The purpose of the research conducted was to see if linear filters usually used signal-processing applications could be effectively applied to portfolio optimization problems in order to reduce the realized risk of that portfolio over a certain period of time. To test this hypothesis, we implemented two iterative algorithms that converge to the minimum variance distortionless response (MVDR) filter. The reason for using this particular filter was that its solution is mathematically equivalent to that of the minimum variance Markowitz portfolio.

Of the two iterative algorithms we implemented, the first was a recursive algorithm that, which yielded a set of potential portfolio weight vectors by computing each successive weight vector by subtracting the product of an auxiliary vector orthogonal to the previous weight vector and a constant that would minimize the output variance of the successive weight vector from the previous weight vector. This algorithm is referred to as the MVDR iterative algorithm. The second algorithm was very similar to the first, except the constants were re-calculated at the end of the algorithm. This is referred to as the Block matrix Algorithm.

The filters return a matrix of potential weight vectors for our portfolio, but do not actually pinpoint the exact weight vector that will yield the lowest realized risk. As such, we tested multiple cross-validation training set sizes to see which training set, on average, identified a weight vector that led to a realized risk that was closest to the minimum achievable realized risk from the filters. We saw that 3-fold cross-validation was the most accurate estimator for the minimum potential realized risk and we recorded the average and standard deviation of the difference between our 3-fold cross-validation estimate and other covariance estimators.

In order to test the efficiency of our method, we looked at multiple factors that could affect its performance. We tested whether the lengths of the sampling window and the prediction window affected how well both of these filters reduced the realized risk of a portfolio over the length of the prediction window. We also tested whether the stock composition of our portfolio affected the realized risk by randomly selecting groups of 50 and 30 stocks from the NYSE 100 and applying the filters to them. We compared the performance of our filters to other portfolio risk estimators: the minimum variance Markowitz portfolio, two spectral estimators derived from Random Matrix Theory, and a single index model that uses the value-weighted return of the S&P 500 to compute the covariance matrix of the portfolio. Because our filters do not alter the covariance matrix of the time-series data, we also investigated whether our filters could be combined with other covariance matrix estimators such as the ones we used for performance testing to further decrease the potential realized risk of our portfolio.

Our results were overwhelmingly encouraging. We found that as we approach the boundary where N (the number of stocks in our portfolio) approaches T (the number of days in our sampling window), our filters perform vastly superior to the minimum variance Markowitz portfolio and single index model, but does not always outperform other the spectral estimators when using the cross-validation estimate. As we increase T, the achieved realized risk is no longer as superior to the Markowitz portfolio, but it, on average, distinctly outperforms every estimator we tested it against. The number of stocks in the portfolio, and which stocks are in it, seem to have no impact on the realized risk we achieve with our filters, as long as the portfolio is well diversified. We also found that our filters could be combined with other estimators to further reduce the realized risk. While this performance increase is marginal at lower values of T, it is noticeable as T increases.